Lesson 11: Chain Rule

CHAIN RULE: $\boxed{\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot \frac{d}{dx}[g(x)] = f'(g(x))\, g'(x)}$

Ex 1 $f(x) = (\sin x)^2$ OUT $= u^2$ IN $= \sin x$ $\frac{d}{du}[u^2] = 2u$

$f'(x) = 2(\sin x)\frac{d}{dx}[\sin x]$

$= \boxed{2\sin x \cos x}$

Ex 2 $y = (4x+1)^3$ OUT $= u^3$ $\frac{d}{du}[u^3] = 3u^2$

$y' = 3(4x+1)^2 \cdot \frac{d}{dx}[4x+1]$

$= 3(4x+1)^2 \cdot 4$

$= \boxed{12(4x+1)^2}$

Ex 3 $h(x) = \frac{1}{(x^2+27)^3}$ OUT $= \frac{1}{u^3} = u^{-3}$ $\frac{d}{du}[u^{-3}] = -3u^{-4}$

$h'(x) = -3(x^2+27)^{-4} \cdot \frac{d}{dx}[x^2+27]$

$= -3(x^2+27)^{-4}(2x)$

$= \boxed{\frac{-6x}{(x^2+27)^4}}$

Ex 4 $y = 3\sqrt{e^x+1}$ OUT $= 3u^{1/2}$ $\frac{d}{du}[3u^{1/2}] = \frac{3}{2}u^{-1/2}$

$y' = \frac{3}{2}(e^x+1)^{-1/2} \cdot \frac{d}{dx}[e^x+1]$

$= \frac{3}{2}(e^x+1)^{-1/2}(e^x)$

$= \boxed{\frac{3e^x}{2\sqrt{e^x+1}}}$

Ex 5 $f(x) = 3\sec^2 x$

$= 3(\sec x)^2$ OUT $= 3u^2$ $\frac{d}{du}[3u^2] = 6u$

$f'(x) = 6(\sec x) \cdot \frac{d}{dx}[\sec x]$

$= 6\sec x(\sec x \tan x)$

$= \boxed{6\sec^2 x \tan x}$

Ex 6 $y = \frac{1}{r^2 + 10x^2}$ where r is a constant

OUT $= \frac{1}{u} = u^{-1}$ $\frac{d}{du}[u^{-1}] = -u^{-2}$

$y' = -(r^2+10x^2)^{-2} \cdot \frac{d}{dx}[r^2+10x^2]$

$= -(r^2+10x^2)^{-2}(0 + 20x)$

$= \boxed{\frac{-20x}{(r^2+10x^2)^2}}$

Ex 7 $g(x) = e^{3x}$ OUT $= e^u$

$g'(x) = e^{3x} \cdot \frac{d}{dx}[3x]$ (OR $g(x) = (e^x)^3$)

$= \boxed{3e^{3x}}$

Ex 8 $h(x) = \cot(3x)$ OUT $= \cot u$ $\frac{d}{du}[\cot u] = -\csc^2 u$

$h'(x) = -\csc^2(3x) \cdot \frac{d}{dx}[3x]$

$= \boxed{-3\csc^2(3x)}$

Ex 9 $y = 3\sin(xe^x)$ OUT $= 3\sin u$

$y' = 3\cos(xe^x) \cdot \frac{d}{dx}[xe^x]$ ← PRODUCT!

$\frac{d}{dx}[xe^x] = 1 \cdot e^x + x \cdot e^x$
$= e^x + xe^x$

$= \boxed{3\cos(xe^x)(e^x + xe^x)}$

Ex 10 $\sin(\sin(e^x))$

Chain rule inside of chain rule!